Introduction to Lie Group and Lie Algebra

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January 15th, 2018, Phnom Penh

Motivation

Earlier in nineteenth century, Évariste Galois (1811-1832) had used group theory to solve algebraic (polynomial) equations that were quadratic, cubic, and quartic. In addition, he was able to prove that no closed form (radical form) solution could be constructed for the general quantic (or any higher degree) equation using only the four standard operations of arithmetic $(+, -, \times, \div)$ as well as extraction of the n^{th} roots of a complex number.

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Theorem (Galois)

A polynomial equation over the complex field is solvable by radical if and only if its Galois group G contains a chain of subgroup
$G = G_0 \supset G_1 \supset \cdots \supset G_\omega = I$ with the property

(1). G_{i+1} is an invariant subgroup of G_i.

(2). each factor group ${}^{G_i}\!\big/_{G_{i+1}}$ is commutative.

Marius Sophus Lie (1842-1899) began his program on the basis of analogy. If finite groups were required to decide on the solvability of finite-degree polynomial equations, then "infinite groups" i.e. groups depending continuously on one or more real or complex variables would probably be involved in the treatment of ordinary and partial differential equations.

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Moreover, Lie knew that the structure of the polynomial's invariance (Galois) group not only determined whether the equation was solvable in closed form, but also provided the algorithm for constructing the solution in the case that the equation was solvable.

He therefore felt that the structure of an ordinary differential equation's invariance group would determine whether or not the equation could be solved or simplified and, if so, the group's structure would also provide the algorithm for constructing the solution or simplification.

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Thus, Lie set the program of computing the invariance group of ordinary differential equation. He also began studying the structure of the children he begat, which we now call **Lie groups**. One of the key ideas in the theory of Lie groups is to replace the global object, the group, with its local or linearized version, which Lie himself called its "infinitesimal group" and which has become known as its **Lie algebra**.

Differentiable Manifolds

Definition

A topological space M is **an n-dimensional locally Euclidean** if every point p in M has an neighborhood U such that there is a homeomorphism ϕ from U onto an open subset of $\mathbb{R}^n$. We call the pair $(U, \phi : U \to \mathbb{R}^n)$ a **chart**, U a **coordinate neighborhood** or a **coordinate open set**, and ϕ a **coordinate map** or a **coordinate system** on U. We say that a chart (U, ϕ) is **center** at $p \in U$ if $\phi(p) = 0$.

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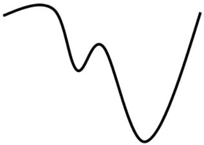
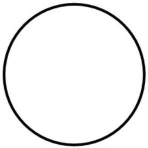
Remark: A Hausdorff, second countable, n- dimensional locally Euclidean space is call a **topological manifold** (with dimension n).

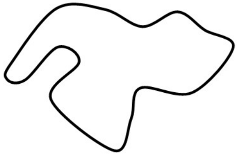
Example: 1-dimensional topological manifolds



Any (simple) curve

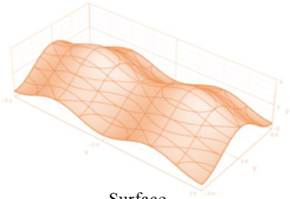


circle

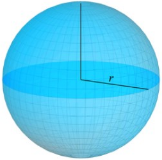


Any (simple) loop

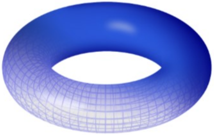
2-dimensional topological manifolds



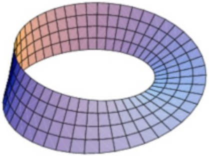
Surface



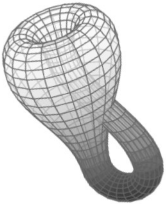
Sphere



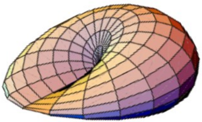
Torus



Mobïus strip



Klein bottle



Cross cap

Definition

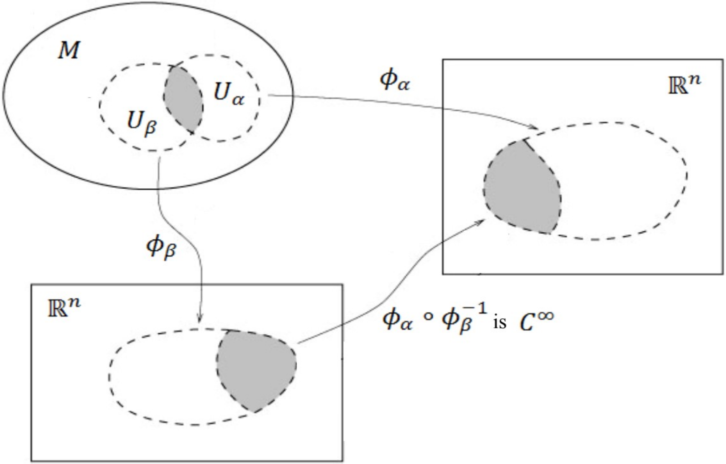
A **differentiable structure** or **smooth structure** on an n-dimensional topological manifold M is a collection of chart $\mathscr{F} = \{(U_\alpha, \phi_\alpha) | \alpha \in I\}$ satisfying the following three properties:

P1: $M = \bigcup_{\alpha \in I} U_\alpha$.

P2: $\phi_\alpha \circ \phi_\beta^{-1}$ is C^∞ for all $\alpha, \beta \in I$ with $U_\alpha \cap U_\beta \neq \varnothing$.

P3: The collection $\mathscr{F}$ is maximal with respect to P2; that is, if (U, ϕ) is a chart such that ϕ and ϕ_α are *compatible* for all $\alpha \in I$, that is, $\phi \circ \phi_\alpha^{-1}$ and $\phi_\alpha \circ \phi^{-1}$ are C^∞ for all $\alpha \in I$, then $(U, \phi) \in \mathscr{F}$.

A pair $(M, \mathscr{F})$ is an n-**dimensional differentiable manifold** or **smooth manifold**.



Differentiable Manifold

Example:
(1). The Euclidean space $\mathbb{R}^n$ with the standard differentiable structure $\mathscr{F}$ that is the **maximal** containing the single chat $(\mathbb{R}^n, id)$, where $id : \mathbb{R}^n \to \mathbb{R}^n$ is the identity map satisfies the properties P1 and P2. Also, $\mathbb{R}^n \setminus \{0\}$ is a differentialble manifold.

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(2). An open set A of a differentiable manifold $(M, \mathscr{F}_M)$ is itself a differentiable manifold. Indeed, if (U_α, ϕ_α) are charts of differentiable manifold M, we define

$$\mathscr{F}_A = \{(A \cap U_\alpha, \phi_\alpha|_{A \cap U_\alpha})|(U_\alpha, \phi_\alpha) \in \mathscr{F}_M\}$$

where $\phi_\alpha|_{A\cap U_\alpha}$ is a restriction of ϕ_α in $A \cap U_\alpha$

then $\mathscr{F}_\alpha$ is a differentiable structure on A.

(3). The unit circle

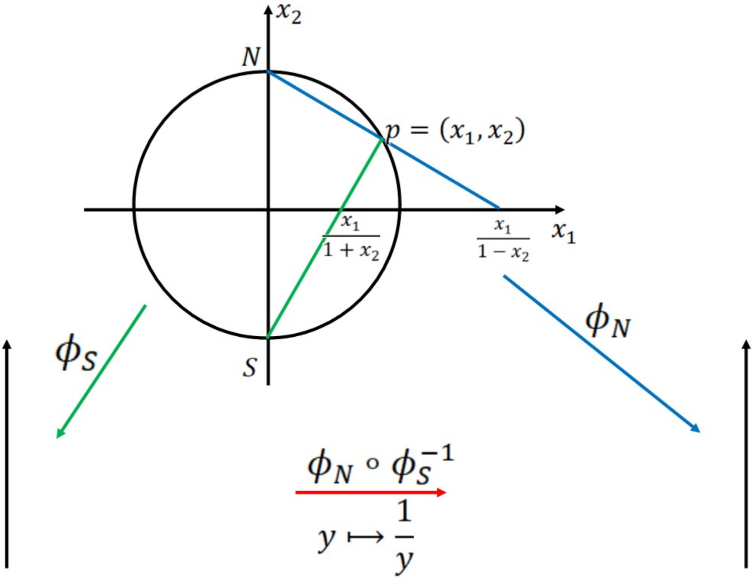
$$S^1 = \left\{ (x_1, x_2) \in \mathbb{R}^2 \middle| x_1^2 + x_2^2 = 1 \right\}$$

is a differentiable manifold. To see this, Let $N = (0, 1)$ be the north pole and $S = (0, -1)$ be the south pole of S^1 and consider the *stereographic projection* from the north pole and south pole

$$\phi_N : S^1 \setminus \{N\} \to \mathbb{R}, (x_1, x_2) \mapsto \frac{x_1}{1 - x_2}$$

$$\phi_S : S^1 \setminus \{S\} \to \mathbb{R}, (x_1, x_2) \mapsto \frac{x_1}{1 + x_2}$$

that take $p = (x_1, x_2) \in S^1 \setminus \{N\}$ (or $S^1 \setminus \{S\}$) into the intersection of the hyperplane $x_2 = 0$ with the line that pass through p and N (or S)


x_2
N
$p = (x_1, x_2)$
$\frac{x_1}{1+x_2}$
$\frac{x_1}{1-x_2}$
x_1
S
ϕ_S
ϕ_N
$\phi_N \circ \phi_S^{-1}$
$y \mapsto \frac{1}{y}$

The maps ϕ_N and ϕ_S are continuous, injective and map onto the hyperplane $x_2 = 0$. The inverse maps

$$\phi_N^{-1}(y) = \left(\frac{2y}{1+y^2}, \frac{y^2-1}{1+y^2}\right)$$
$$\phi_S^{-1}(y) = \left(\frac{2y}{1+y^2}, \frac{1-y^2}{1+y^2}\right)$$

are also continuous. This implies that S^1 is locally Euclidean. Moreover, $S^1 = (S^1 \setminus \{N\}) \cup (S^1 \setminus \{S\})$ and the change of coordinates on $\mathbb{R} \setminus \{0\}$

$$\phi_S \circ \phi_N^{-1}(y) = \phi_S \circ \phi_N^{-1}(y) = \frac{1}{y}$$

is smooth.
Therefore, the maximal contains the chart $\{(S^1 \setminus \{N\}, \phi_N), (S^1 \setminus \{S\}, \phi_S)\}$ is differentiable structure on S^1.

(4). The set of $n \times n$ real matrices $M_n(\mathbb{R})$ and complex matrices $M_n(\mathbb{C})$ are differentiable manifold since we can identifies (linear isomorphic) those sets with $\mathbb{R}^{n^2}$ and $\mathbb{R}^{2n^2}$ respectively.

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(5). From (2), the set of real and complex general linear groups

$$GL(n, \mathbb{R}) = \{A \in M_n(R) | \det(A) \neq 0\} = \overset{-1}{\det}(\mathbb{R} \setminus \{0\})$$

$$GL(n, \mathbb{C}) = \{A \in M_n(C) | \det(A) \neq 0\} = \overset{-1}{\det}(\mathbb{C} \setminus \{0\})$$

where $\det : M_n(\mathbb{R} \text{ or } \mathbb{C}) \to \mathbb{R}(\text{or}\mathbb{C})$ is a determinant map (continuous map), are open sets in $M_n(\mathbb{R})$ and $M_n(\mathbb{C})$ respectively and thus are smooth manifolds.

Definition

The **tangent vector** at $p \in M$ is the linear maps $u_p : \mathscr{D}(M) \to \mathbb{R}$ from the set of real value smooth function on a manifold M ($\mathscr{D}(M)$ or simply $\mathscr{D}$) to a set of real number that satisfies the production rule of derivation. That is, for all $f, g \in \mathscr{D}$ and $\lambda \in \mathbb{R}$,

1. $u_p(f + \lambda g) = u_p(f) + \lambda u_p(g)$ (linearity),
2. $u_p(fg) = u_p(f)g(p) + f(p)u_p(g)$ (product rule of derivation).

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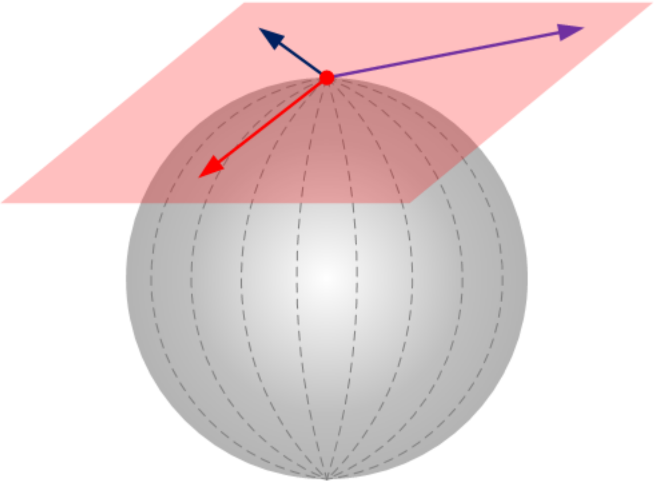
Remark:
The linear map $0 \in T_pM$ and if we define $(u_p + v_p)(f) := u_p(f) + v_p(f)$ and $(\lambda u_p)(f) := \lambda u_p(f)$ for all $u_p, v_p \in T_pM$ and $\lambda \in \mathbb{R}$, then it is easy to see that $(u_p + v_p)(f)$ and $(\lambda u_p(f))$ again are tangent vectors at p. Thus, T_pM is a real vector space.



Tangent space of S^2-sphere

Lie group and its Lie algebra

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Example:

1. The real and complex linear groups

$$GL(n, \mathbb{R}) = \{A \in M(n, \mathbb{R}) | \det A \neq 0\}$$
$$GL(n, \mathbb{C}) = \{A \in M(n, \mathbb{C}) | \det A \neq 0\}$$

The following example are all Lie groups since they are closed subgroup (except 6.) using the following theorem

Theorem (Closed Subgroup Theorem)

Any closed subgroup of Lie group is Lie group.

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Any closed subgroup of Lie group is Lie group.

2. The real and complex special linear groups:

$$SL(n,\mathbb{R}) = \{A \in GL(n,\mathbb{R}) | \det A = 1\}$$
$$SL(n,\mathbb{C}) = \{A \in GL(n,\mathbb{C}) | \det A = 1\}$$

3. The orthogonal group $O(n)$ and the special orthogonal group $SO(n)$:

$$O(n) = \{A \in GL(n, \mathbb{R}) | A^T A = I\}$$
$$SO(n) = \{A \in GL(n, \mathbb{R}) | A^T A = I, \det A = 1\}$$

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4. The unitary group $U(n)$ and the special unitary group $SU(n)$:

$$U(n) = \{A \in GL(n, \mathbb{C}) | A^* A = I\}$$
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5. The Heisenberg group H:

$$H = \left\{ \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \middle| a, b, c \in \mathbb{R} \right\}$$

6. The group
$G = \mathbb{R} \times \mathbb{R} \times S^1 = \{(x, y, z) | x, y, \in \mathbb{R}, z \in S^1\}$ with the group product:

$$(x_1, y_1, z_1) \cdot (x_2, y_2, z_2) = (x_1 + x_2, y_1 + y_2, e^{ix_1 y_2} z_1 z_2)$$

is a Lie group (it is not isomorphic to any matrix groups).

Definition

A **finite-dimensional real or complex Lie algebra** is a finite-dimensional real or complex vector space $\mathfrak{g}$, together with a map $[\cdot,\cdot] : \mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$, which is called a **bracket**, with the following properties:
(1). $[\cdot,\cdot]$ is bilinear.
(2). $[X, Y] = -[Y, X]$ for all $X, Y \in \mathfrak{g}$
(3). $[[X, Y], Z] + [[Z, X], Y] + [[Y, Z], X] = 0$ for all $X, Y, Z \in \mathfrak{g}$

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(2). $[X,Y]=-[Y,X]$ for all $X,Y\in\mathfrak{g}$
(3). $[[X,Y],Z]+[[Z,X],Y]+[[Y,Z],X]=0$ for all $X,Y,Z\in\mathfrak{g}$

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(2). Let $\mathfrak{g} = \mathbb{R}^3$ and define $[x, y]$ to be the cross product $x \times y$. Then $\mathbb{R}^3$ is a real Lie algebra.
(3). The vector spaces $M(n, \mathbb{R})$ and $M(n, \mathbb{C})$ are real Lie algebra and complex Lie algebra, respectively with respect to the bracket operation $[A, B] = AB - BA$.

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Let G be a Lie group.The Lie algebra $\mathfrak{g}$ of G is the **tangent space** T_eG of G at the identity.

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Example:

1. The Lie algebras of $GL(n, \mathbb{R})$ and $GL(n, \mathbb{C})$ are respectively:

$$\mathfrak{gl}(n, \mathbb{R}) = M(n, \mathbb{R})$$
$$\mathfrak{gl}(n, \mathbb{C}) = M(n, \mathbb{C})$$

2. The Lie algebras of $SL(n,\mathbb{R})$ and $SL(n,\mathbb{C})$ are respectively:

$$\mathfrak{sl}(n,\mathbb{R}) = \{X \in \mathfrak{gl}(n,\mathbb{R}) | Tr(X) = 0\}$$
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3. The Lie algebras of $U(n)$ and $SU(n)$ are respectively:

$$\mathfrak{u}(n) = \{X \in \mathfrak{gl}(n,\mathbb{R}) | -X = X^*\}$$
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4. The Lie algebras of $O(n)$ and $SO(n)$ are

$$\mathfrak{o}(n) = \mathfrak{so}(n) = \{X \in \mathfrak{gl}(n, \mathbb{R}) | -X = X^T\}$$

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5. The Lie algebra of the Heisenberg group is the set of 3×3 real upper triangles matrices with zero on the entries of diagonal.

Why Lie groups, Lie algebras?

- Lie group theory establishes surprising relation between many different areas of mathematics, in particular between Algebra, Geometry, Topology, Analysis. For example, The so-called "Fundamental Theorem on maximal tori" in the theory of compact connected Lie groups can be proven
  - algebraically (using Lie algebra techniques)
  - topologically (using the Lefschetz fixed point theorem)
  - geometrically (using the Theorem of Hopf-Rinow)
  - analytically (using integration theory on compact manifolds)

- The study of the irreducible representations of the Lie group $SO(3)$ leads to an explanation of the Periodic Table of the chemical elements.

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- The Lie group "$SU(3) \times SU(2) \times U(1)$" plays a central role in the "Standard Model" of Elementary Particle Physics, which unifies three of the four fundamental forces in nature (electromagnetic, weak, and strong interaction).

- There is correspondence between Lie group and its Lie algebra. This correspondence allows us to study Lie group in term of Lie algebra. Working on Lie algebra which is linear object is a lot easier.

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Thank You!